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TECHNICAL NOTES

An experimental study of laminar heat transfer in a one-porous-wall square duct with suction flow

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INTRODUCTION

The flow and heat transfer characteristics in a duct with wall fluid injection or suction has been investigated extensively for many engineering applications, including solar collectors and fuel cell stacks. The combined laminar flow and heat transfer in porous pipes was studied by Yuan and Finkelstein [1] and that of porous channels was investigated by Terrill [2]. The other related studies can be referred to the review of Kays and Perkins [3].

Hwang *et al.* [4] studied theoretically the developing laminar flow in a one-porous-wall square duct with fluid injection or suction and a constant wall heat flux. Both local friction factors and Nusselt numbers for various Prandtl numbers in the developing and fully developed regions were examined. The friction factors and Nusselt numbers were correlated in Hwang *et al.* [5]. Recently, the theoretical results for injection flow were verified experimentally [6]. In the present investigation, the experimental system [6] has been modified for the cases of suction flow. The mean Nusselt numbers are computed from the measured wall temperatures and deduced bulk mean temperatures. Comparison between the present experimental data and the previous theoretical results, Hwang *et al.* [4, 5], is also made.

PHYSICAL MODEL

Considering a steady air flow in a square duct, the lower duct wall is porous and subjected to a constant heat flux, while the other three walls are impermeable and adiabatic. The entry velocity and temperature profiles of the axial mainstream are uniform. The air flow is withdrawn uniformly from the duct flow through the porous wall. The physical configuration and coordinates are shown in Fig. 1.

As depicted in Hwang *et al.* [4], the flow and heat transfer developments are caused by both the entrance and suction effects. As shown in Fig. 1, the center plane axial velocity profile is normal to the porous wall. Because of the mass withdrawal from the porous wall, the axial velocity is decreased with its peak shifted toward the porous wall. A larger transverse velocity in the lower region of the duct is always observed. The air temperature with higher value in the lower region increases along the flow direction due to the heated porous wall.

Similar to the analysis of Cheng and Hwang [6], the Nusselt number can be determined by three-dimensionless variables, i.e. the dimensionless axial distance, $x^+ = X/(PrDRe_0)$, the Prandtl number, Pr and the wall Reynolds number, $Re_w = V_wD/\upsilon$, as

$$Nu = F(x^+, Pr, Re_w). \tag{1}$$

It is noted that $Pr = 0.72$ for air is used in the present study, thus only two parameters are left in this equation.

The local Nusselt number can be expressed as

$$Nu = \frac{hD}{k} = \frac{qD}{k(\overline{T_w}-T_b)} = \frac{1}{\overline{\theta_w}-\theta_b} \tag{2}$$

where $h = q/(\overline{T_w}-T_b)$ is the local heat transfer coefficient, $\overline{T_w}-T_b$ is the temperature difference of the porous wall and the bulk fluid, q is the heat flux of the porous wall, k is the air thermal conductivity of fluid, and $\overline{\theta_w} = \overline{T_w}/(qD/k)$ and $\theta_b = T_b/(qD/k)$ are the average dimensionless temperatures of the porous wall and the bulk fluid, respectively. In addition, the mean Nusselt number is written as

$$Nu_m = \frac{1}{(\overline{\theta_w}-\theta_b)_m} \tag{3}$$

where the subscript m is the mean temperature difference over an axial distance.

EXPERIMENTAL SYSTEM

The experimental setup used for this study, including an air supply, a settling chamber, a suction unit and a test section, had been described previously, Cheng and Hwang [6]. Instead of air compressor, a 5 hp vacuum pump was installed for fluid withdrawal in the suction unit which was located underneath the test duct. The suction flowrate was regulated by a surge tank and an inverter-controlled motor-pump unit. This suction unit consisted four layers of Bakelite plates with the dimensions 870 × 80 × 50 mm. The sucked air flow first passed three thick layers of ceramic fiber and a graphite plate with 1 mm thickness and then the air flow was conducted to a closed-end tube with a series of downward pin holes.

The lower wall of the test section was a sheet of stainless steel screen (Mesh 325) with area of 20 × 800 mm. The metal screen was heated by a d.c. power supply. Pairs of thermocouples were installed under the screen along the mainstream direction for temperature measurement. The duct dimension was 20 × 20 × 800 mm and the ratio of the axial length to the hydraulic diameter was 40. In addition, mass flowmeters were used for the inlet and outlet flowrate measurements and the suction flowrate was calculated by using the difference of these two flowrates.

DATA REDUCTION

Typically in the experiment, the difference between the inlet and outlet air temperatures was less than 10°C and the

NOMENCLATURE

d	equivalent diameter of the micro-channels of porous wall
D	side length of a square duct
Gr	Grashof number, $g\beta D^3 \Delta T/v^2$
h	heat transfer coefficient, $q/(\overline{T_w} - T_b)$
k	thermal conductivity
Nu	local Nusselt number, hD/k
P	porosity of the porous wall
Pr	Prandtl number, v/α
q	heat flux on the porous wall
Re_0	inlet Reynolds number, U_0D/v
Re_w	wall Reynolds number, V_wD/v
T	temperature of fluid
U, V	velocity components in the X and Y directions
X, Y, Z	rectangular coordinates
x, y, z	dimensionless rectangular coordinates, $X/(DRe_0)$, Y/D, Z/D.

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
η^+	normalized axial coordinate for Nu correlation
θ	dimensionless temperature, $(T - T_0)/(qD/k)$
v	kinematic viscosity.

Subscripts

b	bulk fluid condition
f	fully developed or final developing value
m	average value over an axial distance
s	sucked fluid
w	wall condition
0	inlet condition.

Superscripts

$^-$	cross-sectional average value at a fixed axial location
$^+$	definition for $x^+ = x/Pr$
*	measured wall value.

wall temperature was not higher than 80°C with the inlet bulk temperature of about 25–30°C. The physical properties were evaluated at the average value of the fluid bulk and wall temperatures at each specific axial distance.

Due to the porosity of the screen heater of the lower wall, the wall temperature was not equal to that of screen heater. To remedy this discrepancy, the wall temperature can be computed by

$$\overline{T_w} = \overline{T_w^*}(1-P) + T_sP, \tag{4}$$

where $\overline{T_w^*}$ is the measured mean wall temperature, T_s is the sucked air temperature and P is the porosity of the porous wall and equals to 0.345 in this study. Considering the micro-structure of the porous wall as micro-channels, the sucked air temperatures T_s can be determined by using the definition, $T_s = \overline{T_w^*} - (qd)/(kNu)$ where $d = 0.044$ mm is the equivalent diameter of the micro-channels. The solution proposed by Hsu [7] may be used for evaluation of Nu with the known Prandtl number, the Peclet number and the length of the micro-channels (0.030 mm). The method of deduction of the bulk mean air temperatures from the measured data is taken from Cheng and Hwang [6]. According to equation (3), the mean Nusselt numbers can be obtained from the mean difference values of porous wall and bulk fluid temperatures.

For ensuring the condition of laminar flow in the duct, all of the flow in the test section were kept from the point of flow separation or complete mass extraction. The flow separation and mass extraction may induce turbulent flow as reported by Hwang *et al.* [4]. Moreover, the value of Gr/Re^2 in the test is only about 0.03 for the smallest inlet Reynolds number, 550, and the largest temperature difference, 50°C, between the top and bottom wall at the exit of test section. Thus, the forced convection is dominant in heat transfer for the experiment and the gravitation field is not important. The present results may be also applied to the case with upper wall suction.

The experimental uncertainty analysis of this study is similar to that of the authors' previous paper [6]. Because all of

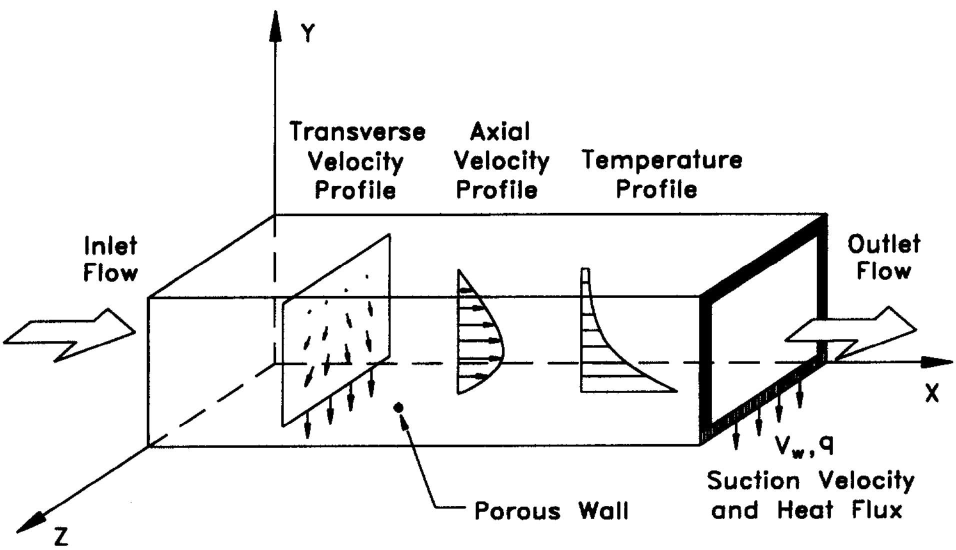

Fig. 1. Physical configuration and coordinates.

the variations of flow velocity, temperature and heating rate fall in the same range of the previous work, the largest estimated uncertainty of Nu_m is not more than 10% for all cases in this study.

RESULTS AND DISCUSSION

For obtaining the average wall temperature $(\overline{\theta_w})_m$ and the bulk fluid temperature, $(\theta_b)_m$, along the duct, experiments were conducted in the range of inlet Reynolds numbers $Re_0 = 550$, 600, 700, 800, 1000, 1200, 1500 and 2000 with various wall Reynolds numbers $Re_w = 0$, -2 and -5. As shown in Fig. 2, the average dimensionless temperature of the porous wall is always higher than that of the bulk fluid. Due to the effect of suction flow, the wall temperatures at a fixed axial location decrease with the suction rate. For a fully developed flow without suction $Re_w = 0$, the variation of bulk mean temperature is linear. The bulk mean temperature at a fixed axial location is reduced by the air extraction. According to the results shown in this figure, the agreement between the experimental values and the theoretical ones is fairly good for all the wall Reynolds numbers.

The effect of Re_w on the deduced Nusselt numbers along the axial direction for $Pr = 0.72$ is given in Fig. 3. It is noted that heat transfer is enhanced by the fluid suction and Nu_m is increased with the suction rate. As can be seen in the figure, the experimental data of Nu_m agree with the previous theoretical results, Hwang *et al.* [4]. In the present study, the experimental data of the mean Nusselt numbers can be correlated as

$$\frac{Nu_m}{Nu_f} = 1 + \frac{0.0954}{\eta^+} \tag{5}$$

where Nu_f is the fully developed Nusselt number for $Re_w = 0$ and -2 and is the final developing value before complete extraction for $Re_w = -5$. As indicated by Hwang *et al.* [5], the values of Nu_f are 2.71, 3.51 and 5.06 for $Re_w = 0$, -2, and -5, respectively. In addition, η^+ is the normalized axial coordinate and is defined as

$$\eta^+ = \frac{(x^+)^2}{A(x^+) - 0.129B(x^+)^{0.5} + 0.00589C} \tag{6}$$

where the coefficients in the denominator are

$$A = 1 + 0.142Re_w - 0.00421Re_w^2$$

$$B = 1 + 0.162Re_w - 0.00303Re_w^2$$

$$C = 1 + 0.185Re_w - 0.00223Re_w^2. \tag{7}$$

The results of equation (5) are plotted in Fig. 4. It is seen that the value of Nu_m/Nu_f is decreased monotonically with the increase in η^+, and a limiting fully developed value of $Nu_m/Nu_f = 1$ for large η^+ can be expected. The present experimental data are deduced in this figure for comparison. The dashed lines show the curves with deviations of $\pm 15\%$ from equation (5). More than 95% of experimental data are within $\pm 15\%$ of the correlated curve for the entire ranges of η^+ and Re_w under study.

CONCLUSIONS

(1) The measured porous wall temperatures, bulk fluid temperatures and Nusselt numbers agree fairly well with the theoretical results.

(2) It is verified experimentally that the Nusselt number is higher for a larger suction rate. More than 95% of the deduced Nusselt numbers are within $\pm 15\%$ of the correlation equation for the entire ranges of η^+ and Re_w under study.

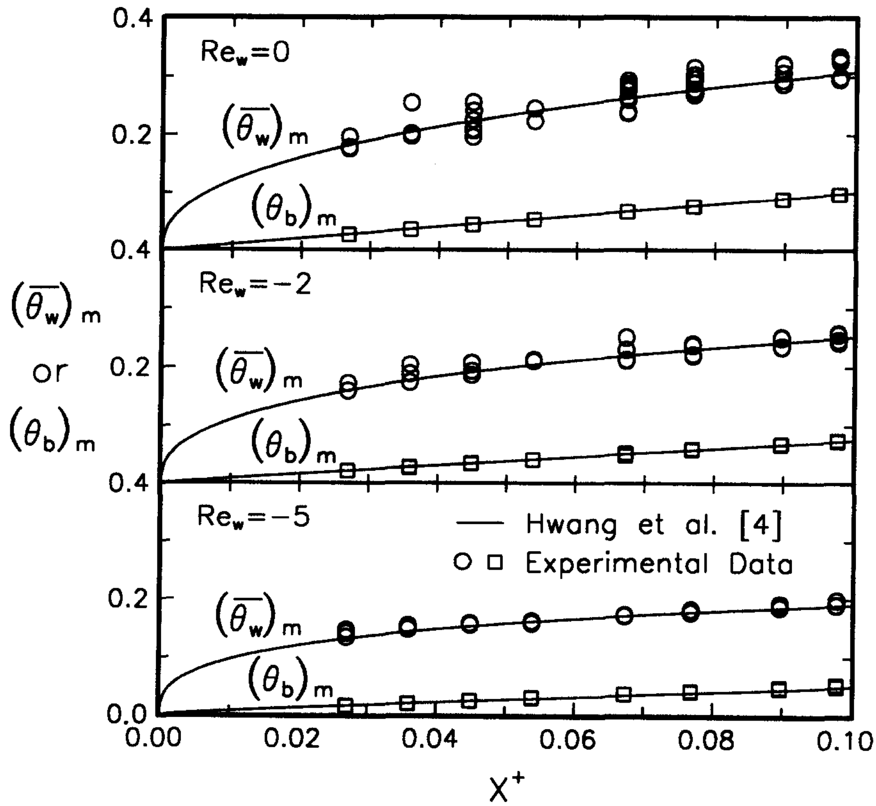

Fig. 2. Porous wall and bulk fluid temperatures along the axial length for various wall Reynolds numbers.

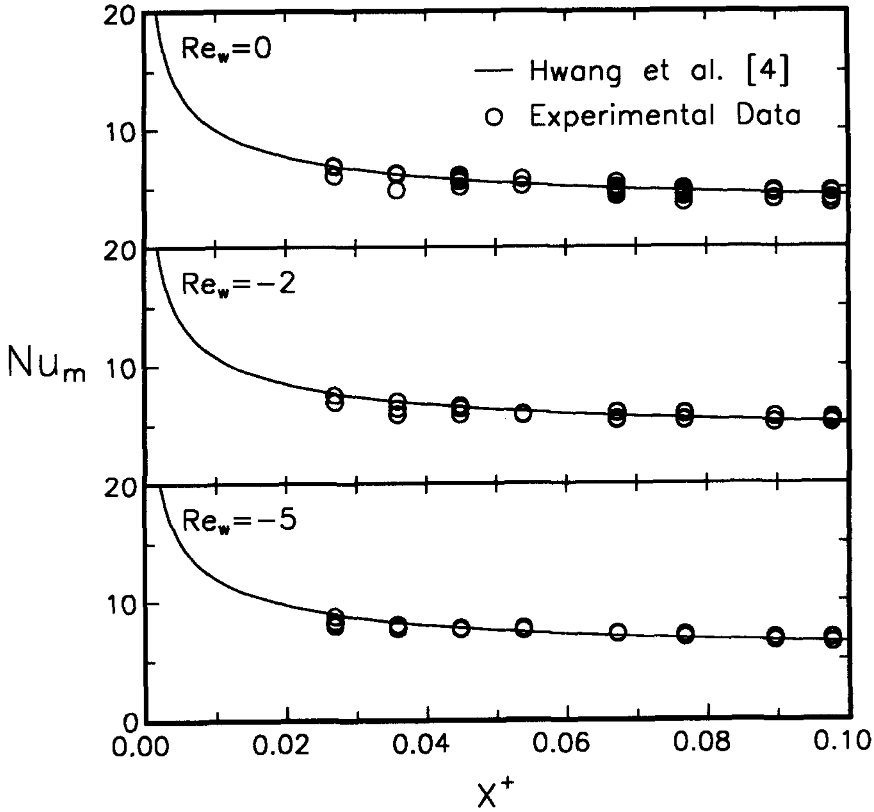

Fig. 3. Nusselt numbers along the axial length for various wall Reynolds numbers.

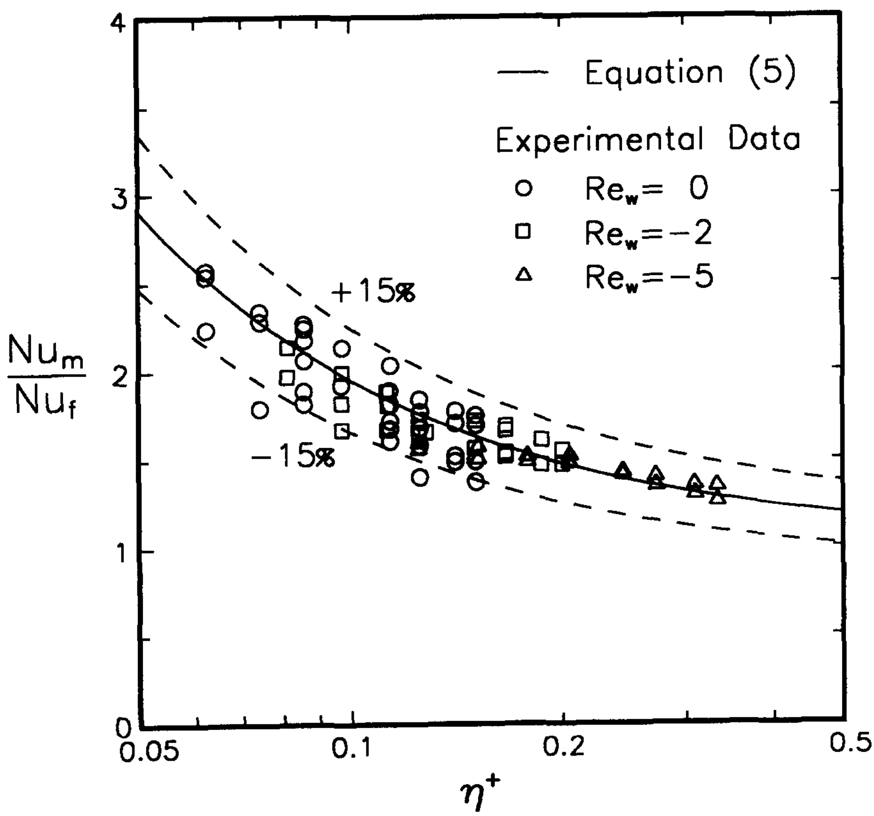

Fig. 4. Comparison of experimental data with correlation results of Nusselt numbers.

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Natural convection in porous media near L-shaped corners

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1. INTRODUCTION

Natural convection in porous media has applications such as building insulation, underground energy and geophysical systems. Since the similarity analysis of Cheng and Minkowycz [1] on natural convection from a vertical surface, numerous studies have been reported in the literature [2]. However, natural convection heat transfer from L-shaped corners in porous media, which has geophysical and technological applications has received much less attention.

Daniels and Simpkins [3] studied natural convection in the region bounded by a uniformly heated vertical wall and a thermally insulated wall which forms a corner of arbitrary angle. Hsu and Cheng [4] considered a semi-infinite inclined heated surface, attached to another unheated surface extending upstream at an arbitrary angle. In both of these investigations [3,4], the method of matched asymptotic expansions was used. Related problems of natural convection from vertical corners placed in a porous medium have also been investigated [5–7].

In the current work, numerical solutions for natural convection in a Darcian fluid confined in the region of a horizontal, L-shaped corner are presented. The corner is formed by a heated isothermal vertical plate joined to a horizontal surface, which is either adiabatic, or held at ambient temperature. The aspect ratio range considered (i.e. the length of the horizontal side/height of the vertical side) falls between the two asymptotic limits of no horizontal wall, and a 'long' wall.

2. ANALYSIS

Steady, two-dimensional (2D) natural convection of a Darcian fluid from an L-shaped corner is considered as shown in Fig. 1, where the coordinate system and the boundary conditions in non-dimensional form are marked. The saturated porous medium is treated as a continuum, with the solid and fluid phases in local thermodynamic equilibrium. The governing equations can be reduced to the following coupled differential equations for the velocity components and temperature in nondimensional form

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